

# Fast Estimation for Electromigration Nucleation Time Based on Random Activation Energy Model

Jingyu Jia, Jianwang Zhai<sup>✉</sup>, Kang Zhao

School of Integrated Circuits, Beijing University of Posts and Telecommunications, China

{jiajingyu, zhajiw, zhaokang}@bupt.edu.cn

**Abstract**—Electromigration (EM) has attracted significant interest in recent years, because the current density of on-chip power delivery networks (PDNs) is always increasing. However, the EM phenomenon is affected by the randomness of the annealing process during nanofabrication, which requires more reliable statistical models for EM analysis. In this work, we propose a fast estimation method for EM nucleation time based on the random activation energy model. Experiments demonstrate that our method can accurately and efficiently analyze the nucleation time distribution under random processes, and achieve 39.1% improvement in estimation speed compared with the previous work.

## I. INTRODUCTION

Under sustained current stress, EM occurs and voids can form in the metal wires, leading to reliability problems. Therefore, it is important to estimate the EM void nucleation time (denoted as  $t_{nuc}$ ). However, PDN fabrication has stochasticity and the number of interconnect tree nodes of very large-scale integration (VLSI) circuits are enormous, resulting in computational difficulties in EM simulation.

Many EM analysis methods have been developed to address the challenges. Korhonen *et al.* [1] propose the Korhonen's equation to model the EM stress distribution on metal wires. Cook *et al.* [2] introduce a finite difference method (FDM), a representative numerical method, but it suffers from low efficiency. Wang *et al.* [3] present an efficient eigenfunction-based method, an example of analytical methods, but still lacked a more accurate solution for the random EM phenomenon. Previous works [4], [5] consider the random factors of EM and solve it using numerical methods, *i.e.*, linear time-invariant system and FDM, which are less efficient.

To solve these problems, we propose a fast estimation method for EM nucleation time based on random activation energy model. The main contributions are as follows:

- We select typical atom activation energy based on the random activation energy model to quickly identify the key interconnect trees that determines the  $t_{nuc}$ .
- We perform Monte Carlo sampling according to the random activation energy distribution, to analyze the  $t_{nuc}$  by simulating the realistic EM scenarios.
- We exploit the independence of the interconnect trees and conduct parallel analysis on the key interconnect trees, which improves the efficiency of EM analysis.

<sup>✉</sup> Corresponding author. This work is supported in part by Beijing Natural Science Foundation under Grant 4244107 and National Key R&D Program of China (2022YFB2901100).

## II. RANDOM ACTIVATION ENERGY MODEL

The randomness of the grain microstructure such as grain sizes leads to a random distribution of the metal atom activation energy  $E_a^{eff}$ , which is one of the key factors affecting EM. Previous work in [6] explored the impact of random activation energy on EM, but it is inefficient. The numerical value of  $E_a^{eff}$  follows a normal distribution:

$$f(E_a^{eff}) = \frac{1}{\bar{\sigma}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{E_a^{eff} - \overline{E_a^{eff}}}{\bar{\sigma}}\right)^2\right], \quad (1)$$

where  $\bar{\sigma}$  is the standard deviation of  $E_a^{eff}$ , and  $\overline{E_a^{eff}}$  is the mean value. The specific parameter values depend on the actual technology of on-chip PDNs.

Our method relies on modeling random activation energy  $E_a^{eff}$  that varies across different parts of the interconnect tree. By the central limit theorem,  $E_a^{eff}$  in each interconnect branch is assumed to follow a normal distribution, and its mean value is taken as the representative value in that branch.

## III. FAST ESTIMATION FOR NUCLEATION TIME

The overall flow of our method is illustrated in Fig. 1. First, steady-state stresses are solved to filter out EM-immortal interconnect trees and select key trees for EM analysis. Then, these key trees are Monte Carlo sampled and the  $t_{nuc}$  distribution under the random technology is estimated using the eigenfunction-based method and binary search. Finally, parallel computing is used to accelerate the EM estimation.

### A. Filtering and Selection of Interconnect Trees

There are some interconnect trees whose stress cannot exceed the critical stress even when the stress attains the maximum and stays in steady-state, *i.e.*, these trees are immortal to EM [7]. Therefore, we can filter out these immortal trees by pre-computing the steady-state stress using the method in [8]. As stated in Section II, the atom activation energy  $E_a^{eff}$  follows a normal distribution. The  $E_a^{eff}$  corresponding to the probability peak is the typical value of  $E_a^{eff}$ . We use the eigenfunction-based method [3] and binary search to obtain typical  $t_{nuc}$  for each interconnect tree at this typical value of  $E_a^{eff}$ . Then, we sort all interconnect trees according to typical  $t_{nuc}$  and select the partial of trees with smaller  $t_{nuc}$  as the key interconnect trees of the whole PDN. This method can reduce the computational effort of EM-immortal trees that will not generate voids, effectively accelerating the  $t_{nuc}$  estimation under random parameters.

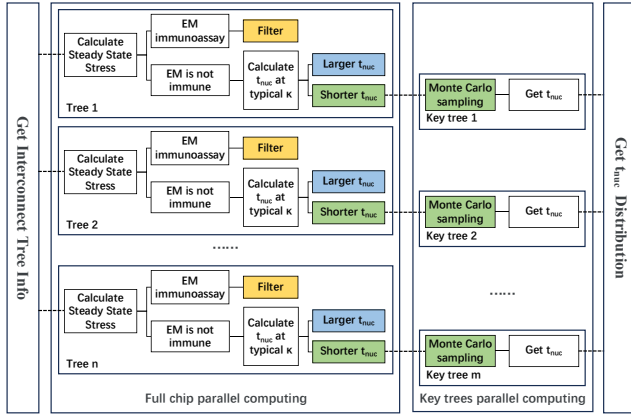


Fig. 1 Illustration of the proposed EM estimation method.

### B. Monte Carlo Sampling

Modeling the whole PDN using traditional methods based only on a specific atom activation energy is highly inaccurate and difficult to verify reliability before mass production. We propose to conduct statistical  $t_{nuc}$  estimation based on Monte Carlo sampling of  $E_a^{eff}$ . In each sampling, the internal nodes of a single branch are assigned randomly sampled  $E_a^{eff}$  according to Section II. The  $E_a^{eff}$  of nodes connecting different branches needs to be expressed according to the boundary conditions of “extended Korhonen’s model”(EKM) proposed in [9], [10]. After Monte Carlo sampling, the eigenfunction-based method [3] is utilized to solve for the transient stress in the interconnect tree according to:

$$\sigma(x, t) = \sigma(x, \infty) - \sum_{m=0}^{\infty} C_m^t \Psi_m(x), \quad (2)$$

where  $\sigma(x, t)$  is the transient stress at time  $t$  and position  $x$ ,  $\sigma(x, \infty)$  is the steady-state stress which is not affected by random activation energy [8],  $\Psi_m$  and  $C_m^t$  are a series of corresponding eigenfunctions and coefficients [3].

We perform a binary search on the time interval to obtain the moment when the maximum value of transient stress reaches the critical stress  $\sigma_{crit}$ , which is  $t_{nuc}$ . When repeated sampling multiple times, we can get the  $t_{nuc}$  distribution.

### C. Parallel Acceleration

When performing EM analysis, it only depends on the voltage, resistance, physical parameters, *etc.* of each interconnect tree itself. Therefore, the  $t_{nuc}$  estimation of the PDN is performed based on each single interconnect tree. We use parallel computing, *i.e.*, OpenMP, for multi-core CPU parallel programming, to calculate the  $t_{nuc}$  of multiple interconnect trees at the same time, thus speeding up the EM analysis.

## IV. EVALUATION

### A. Experimental Setting

Our method is implemented using C++ and Python, and tested on a Linux platform with an 8-core CPU @3.20GHz and 16GB memory. The IBMPG dataset [11] is used as the PDN benchmark. ISPD’23 [5] investigates the effects of random factors such as activation energy and critical stress

TABLE I Statistical Results for Nucleation Time Estimation

Benchmark	# Nodes	# Trees	ISPD’23 [5] (s)	Ours (s)	KL divergence
IBMPG2	127238	462	2868	1710	0.0310
IBMPG3	851584	8189	23245	4856	0.0118
IBMPG4	953583	9641	27496	24379	0.0072
IBMPG5	1079310	1982	9197	7329	0.0012
Avg.	-	-	15701.5	9568.5	0.0128

at non-uniform temperatures and solves Korhonen’s equation using FDM, which served as the baseline. For a fair and accurate comparison, the implementation of ISPD’23 only considers random activation energy. We perform 50 samplings and estimations using both two methods, and selects 10% as the key interconnect trees in ours. The Kullback-Leibler (KL) divergence is used to measure the similarity of the  $t_{nuc}$  distributions considering EM randomness.

### B. Accuracy and Efficiency of Nucleation Time Estimation

TABLE I gives the runtime and KL divergence of  $t_{nuc}$  distribution, where “# Nodes” and “# Trees” are the number of nodes in PDN circuit and interconnect trees respectively. The average KL divergence is 0.0128, demonstrating that our method and ISPD’23 [5] yield highly similar distribution results. Our method improves the estimation speed by 39.1% on average in EM estimation compared with the baseline. Since the number, shape, and physical parameters of the interconnect trees are different, the improvement on each PDN is also different. Overall, Our method is more efficient and can be extended for different large-scale PDNs with better scalability and generalizability.

## V. CONCLUSION

Experiments show that our EM nucleation time estimation method is accurate and more efficient, providing an effective solution for the random problem of EM analysis.

## REFERENCES

- [1] M. A. Korhonen *et al.*, “Stress evolution due to electromigration in confined metal lines,” *Journal of Applied Physics*, vol. 73, no. 8, pp. 3790–3799, 1993.
- [2] C. Cook *et al.*, “Finite difference method for electromigration analysis of multi-branch interconnects,” in *Proc. SMACD*, 2016, pp. 1–4.
- [3] X. Wang *et al.*, “Fast physics-based electromigration analysis for full-chip networks by efficient eigenfunction-based solution,” *IEEE TCAD*, vol. 40, no. 3, pp. 507–520, 2021.
- [4] F. N. Najm *et al.*, “Efficient simulation of electromigration damage in large chip power grids using accurate physical models,” in *Proc. IRPS*, 2019, pp. 1–10.
- [5] A. Kteyan *et al.*, “Electromigration assessment in power grids with account of redundancy and non-uniform temperature distribution,” in *Proc. ISPD*, 2023, pp. 124–132.
- [6] V. Sukharev *et al.*, “Experimental validation of a novel methodology for electromigration assessment in on-chip power grids,” *IEEE TCAD*, vol. 41, no. 11, pp. 4837–4850, 2022.
- [7] S. X.-D. Tan *et al.*, “Long-term reliability of nanometer VLSI systems,” *Cham: Springer*, 2019.
- [8] Z. Sun *et al.*, “Fast electromigration immortality analysis for multi-segment copper interconnect wires,” *IEEE TCAD*, vol. 37, no. 12, pp. 3137–3150, 2018.
- [9] S. Chatterjee *et al.*, “Fast physics-based electromigration checking for on-die power grids,” in *Proc. ICCAD*, 2016, pp. 1–8.
- [10] —, “Power grid electromigration checking using physics-based models,” *IEEE TCAD*, vol. 37, no. 7, pp. 1317–1330, 2017.
- [11] S. R. Nassif, “Power grid analysis benchmarks,” in *Proc. ASPDAC*, 2008, pp. 376–381.